Invited Lecture

What Matters for Effective Mathematics Educator: Preservice or In-service Training?

Alphonse Uworwabayeho[1]

ABSTRACT According to UNESCO (2015), the equity gap in education is exacerbated by the shortage and uneven distribution of professionally trained teachers, especially in disadvantaged areas. Target 4.c (MOI) of the SDG 4 is therefore aimed at substantially increasing the supply of qualified teachers, including through international cooperation for teacher training in developing countries, especially least developed countries and Small Island developing States by 2030. It further states that teachers are one of the fundamental conditions for guaranteeing quality education and therefore there is need to empower and adequately recruit, remunerate and motivate professionally qualified teachers and educators, and support them within a well-resourced, efficient and effectively governed systems (UNESCO, 2015). The knowledge of teachers in the last three decades was mainly influenced by a well-known scholar Lee Shulman who categorized teacher knowledge into seven categories among which content knowledge is included. However, much research on in-service teachers focused on the pedagogical content knowledge hypothesizing the mastery of content as much as they are graduated from recognized training institutions. Based on this categorization, the present paper presents an analysis of Rwandan mathematics school subject leaders' Content Knowledge (CK). The presentation is based on partnership established between governmental and nongovernmental institutions led to development and implementation of certified Continuous Professional Development (CPD) programs for primary and secondary mathematics teachers. Findings reveal a lack of teachers' preparedness to adopt the new curriculum teaching approaches, there is also lack of appropriate physical facilities in schools to accommodate every leaner's individual needs among other hindering factors. Recommendations include systematic CPD programs for in service teachers to complement preservice training so that they can adapt various reforms and in-service teachers to establish their individual professional development plans.

Keywords: Continuous professional development; In-service training; Teacher education.

1. Introduction

It is not doubtable that teaching is a complex work and pre-service teacher education is rarely sufficient to provide all knowledge and skills necessary to successful teaching and students learning (Killen, 2015). Therefore, a significant portion of teacher education can be acquired only on the job. In particular, growing research (e.g., Killen,

[1] University of Rwanda-College of Education. E-mail: a.uworwabayeho@ur.ac.rw

2015; Ball et al., 2008) shows that quality induction has a positive impact on beginning teachers' motivation and commitment; beginning teachers' job satisfaction; beginning teachers' teaching practices and learning outcomes of students. Rwanda registered an impressive growth enrolment especially at primary level as result of different education policies especially with the aim of achieving education for all goals. However, the quality was undermined as demonstrated by learners' low performance from national tests especially in mathematics subject. The low performance was attributed to different factors including lack of teaching and learning materials including textbooks, large class size, heavy teaching load for teachers (Mineduc, 2015). This situation was worsened by the introduction of the competence-based curriculum (CBC) at the beginning of 2016 in primary and secondary education. Since then, different strategies to improve students' performance in mathematics and sciences have been an important concern to each level of education system in Rwanda for all stakeholders. These strategies include teachers training on the implementation of CBC, dissemination of books and laboratory equipment, smart classrooms and improving students' welfare by introducing school feeding program across the whole country. However, as per our own experience, it has been observed that most of strategies set by most of educational stakeholders tend to focus on empowering in-service teachers with pedagogical skills with little, if any, focus on improving teachers' subject content knowledge. Though education statistics affirm that 93.6% of primary school teachers are qualified (Mineduc, 2018), there is a little information on link between pre-service training and mathematics syllabus requirements as well as how teachers are motivated to adapt regular changes in terms of content. In this line, the Rwandan Teacher Development and Management (TDM) Policy states that all beginning teachers — defined as teachers in the first three years of their career, have to receive systematic professional support from their head teachers, subject school leaders, school-based mentors, and school inspectors. For the purpose of the present paper, we limit the description on the role of mathematics subject school leaders (MSSLs). MSSLs are the experienced teachers appointed by the school head teachers to support new qualified mathematics teachers through mentoring and coaching process in addition to their ordinary teaching load (REB, 2019). They are called to play key role in leadership of mathematics teaching and learning in their schools for students' achieving learning outcomes. To assure this important role, they are supposed to be expert in mathematics content knowledge, pedagogical content knowledge, knowledge of mathematics curriculum and its requirements such as ensuring inclusive education in mathematics lessons (making sure all students can learn). Since none of MSSLs was prepared to assume these new responsibilities, it is worthy to interrogate ourselves to what extent selected MSSLs are knowledgeable in terms of curriculum syllabus content. This will inform policy makers and educational partners further trajectories for teacher professional development. We review the existing literature on teachers content knowledge with focus on the work of Shulman (1986), the methodological considerations, discussions of findings and conclusions.

2. Literature Review

Research (Darling-Hammond, 2000; Ingvarson et al., 2005) found that students' mathematical achievement would be attributed to teachers and their teaching practices; though more teacher mathematical knowledge does not necessarily imply more student learning (Ball, Loewenberg, Thames, & Phelps, 2008). In other words, having more knowledge of mathematics does not automatically lead to better teaching of mathematics. Shulman (1986) identified different types of knowledge that are required for being effective teacher: mathematics content knowledge, pedagogical content knowledge and curriculum knowledge. Later work in mathematics education built on Shulman's work (e.g., Ball et.al., 2008) made further distinctions within mathematics content knowledge and pedagogical content knowledge. Inside of subject matter knowledge, it is distinguished between common content knowledge (what most people will know in mathematics) and specialized knowledge (what people who have studied mathematics will know) (Ball et al., 2008). In other words, subject matter knowledge is knowledge of mathematics. It's not about knowing about children or how to teach the mathematics to children.

Concerning with pedagogical content knowledge, there are mixtures of knowing the mathematics content and knowing how to teach it. It involves knowledge of the mathematics curriculum that lies ahead and how what is taught has an impact on children's learning of mathematics in their later life. Knowledge of content and curriculum is about what is taught when; and how do learners move through topics. If you are a mathematician, it is quite unclear how things are ordered and why. It is knowledge of when to introduce a concept and how each concept builds on previous knowledge and forms a building block to more advanced knowledge. This overlaps with the knowledge of students and knowledge of content and teaching. For example, a teacher who plans to teach a lesson on multiplying decimals needs to know a lot more than how to do the multiplication: *"The teacher had to know more than how to multiply decimals correctly herself. She had to understand why the algorithm for multiplying decimals works and what might be confusing about it for students."*(Ball, 1990; p. 448). In the first part of the statement, it is the content knowledge for mathematicians while the second concerns with mathematics content knowledge for teachers. Research in developed countries illustrates that a substantial part of the difference in student achievement is attributable to teachers and their teaching practices (Darling-Hammond, 2000; Rice, 2003, Ingvarson et al., 2004). In developing countries, and with specific regard to Lesotho, it was found that teacher characteristics such as gender, class size or years of experience had no influence on students' mathematical achievement (Parke and Kanyongo, 2012); rather teachers' content knowledge seems to be the only certain influence on students' mathematical achievement. Based on the argument that the teacher needs to know more about the topic he/she has to teach, the paper focuses its investigation on in-service teachers' subject matter knowledge in different areas of the mathematics syllabus in place.

3. Methodological Considerations

As mentioned throughout previous sections, the paper concerns with analysis of mathematics subject school leaders (MSSLs) content knowledge. The motivation for this paper draws appointment of these teachers to implement professional development strategies put in place by the Rwanda ministry of education. On one hand, we argue that success for these strategies depends on competencies of MSSLs for the effective delivery of the desired outcomes. On the other hand, we know that none of these MSSLs was prepared to assume these responsibilities added to their normal teaching load. Therefore, this becomes a challenge for not only themselves but also for education local education leaders in terms of the implementation. It is within this context, a partnership between Rwanda Education Board (Rwanda governmental organ in charge of implementation education policies up to secondary school level), the University of Rwanda-College of Education (the unique Rwanda government higher learning institution in charge of education training) and the Flemish Association for Development Cooperation and Technical Assistance (VVOB) organized a formal credited continuous professional development program for MSSLs. The program introduces MSSLs to a variety of aspects of pedagogical content knowledge for teaching mathematics and subject leadership. The overall aim of the program is to equipping them with mentoring and coaching skills of their fellows and new qualified teachers. In this way, to support the implementation of the CBC, the training is also competence-based whereby the focus is on practice based and learning collaboration. Training approaches includes learning collaboration through developing culture of discussion, active participation and micro teaching. In order to make microteaching relevant and meaningful for participants, facilitators judged it better to focus as much as possible on mathematics topic area that seem to be difficult to introduce to learners. But how could we decide which topic was the most difficult? One way was to ask participants to name these topics; but this would lead to individuals' sentiments. Hypothetically, once one does not master a given topic, he is likely either to avoid teaching it or badly teach it, thus students' missing part of the curriculum content. We therefore preferred to use a simple test whereby participants were given a series of questions covering the six topic areas (number and operations, fractions and proportional reasoning, metric measurements, geometry, algebra, statistics and elementary probability) as identified in the national mathematics syllabus (REB, 2015a, p.18). Therefore, the purpose of the test was not evaluating participants' mathematics content knowledge; rather supporting facilitators identifying topics of the primary mathematics curriculum to be used in the training process.

Though the program has to reach all MSSLs in six districts that registered learners' low performance in national mathematics examination ending primary education in addition to high rate of dropouts, the present study concerns 39 teachers (one MSSL per primary school in six selected districts) who were invited to starting the CPD program. It is worthy to mention that an MSSL is appointed by the school leader on the basis to have served at least 3 years in the same school and all MSSLs of the six

districts have to benefit of the CPD program at different cohorts. These 39 MSSLs comprised of 8 females and 31 males. They are in range of 5 to 30 years of teaching experience.

The mathematics test took place on the first day of the actual training that consists in face to face sessions in three different centers with two facilitators from URCE per centre. Prior to administer the mathematics test, facilitators (authors of the paper) briefed MSSLs on modalities governing training and the rationale of the training and the test. MSSLs signed individual consent form related to video recording, interviews and any other type of data with educational purpose. It is expected that some videos of good practices may be shared for educational purpose. Participants had one after another two question papers on mathematics content and pedagogical content knowledge; reasonable period of 2hours for each component. But the focus of the paper is mathematics that was composed of 20 mixed open and multiple-choice questions. Answer sheets were marked, outcomes analyzed and mean and standard deviation calculated. Results are hereafter graphically or tabularly presented according to the six topic areas from the primary national mathematics syllabus.

4. Results and Discussions

All 20 questions were grouped under six themes that coincide with the six topic areas. One part presents results as a whole group, that is no consideration of district, while another part takes district into account. The rationale behind the second arrangement consists in depicting any contextual particularity of a given that from the 6 districts whereby 4 (Kirehe, Kayonza, Gatsibo and Nyagatare) are located in eastern province and 2 others (Rusizi and Nyabihu) in western province.

4.1. Topic area 1: numbers and operations

The topic on numbers and operations is taught from primary 1 (P1) to primary 6 (P6). Specifically, by the end of primary, learners are expected to be able to read, write and compare whole numbers beyond 1,000,000. In this view, mathematics subject school leaders (MSSLs) were tested on writing numbers in figures. They were also asked to write in words as they would say numbers. Findings on these questions are summarized in Fig. 1. (see in the next page)

4.1.1. Whole numbers

Fig. 1 shows that writing numbers in figures from words was less confusing than writing them in words from figures. For example, all MSSLs (100%) could write correctly “two thousand and fifty in figures” whereas, 2% of them failed to write 140,000 in words. It also shows that writing numbers in figures seemed difficulty for larger numbers, e.g., 10% of MSSLs could not correctly write “*four hundred thousand and seventy-three”* in figures. In addition, writing decimal numbers in words seemed

difficult to some MSSLs (12%). Looking into different answer sheets, it can be observed that some SSLs don't have skills to translate between written decimal numbers and spoken English. Since, being able to write, read and compare numbers is one of targeted key competences for learners in primary schools, teachers of mathematics should be able to help learners achieve this competence (REB,2015). Therefore, there is a justifiable need to involve MSSLs in activities where they work together to understand and adopt appropriate ways of teaching and learning whole numbers.

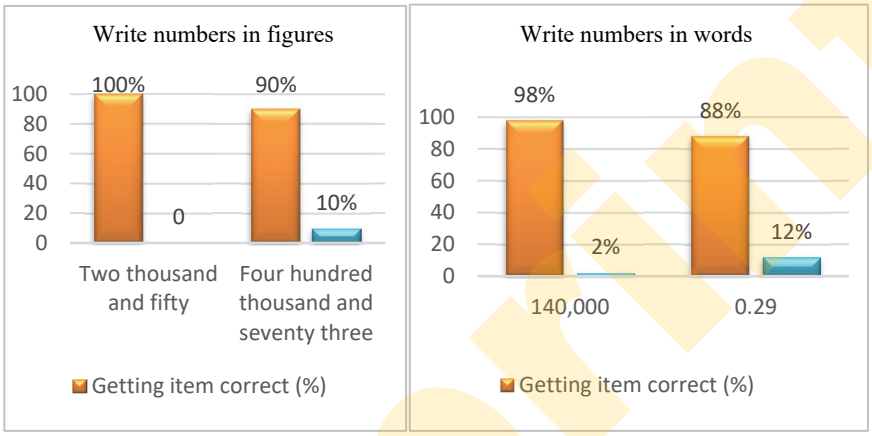

Fig. 1. Write numbers in figures and in words

4.1.2. Decimal Numbers

The national mathematics competence-based syllabus under implementation in Rwanda prevails teaching decimal numbers in upper primary (REB,2015a). From P4 through P6, learners should be able to add, subtract, and compare decimal numbers using place values of decimals up to some numbers of decimal places. Fig. 2 (on the next page) summarizes MSSLs competences to write decimals.

Fig. 2 (a) indicates challenges in writing decimal for some MSSLs whereby 76% could not write correctly "*eleven tenths"* in figures. The majority of those who failed could confuse it with "eleven thousandths" or "eleven hundredths". Other MSSLs could not identify the place values for eleven tenths. On the other side in Fig. 2 (b), 48% of MSSLs failed to compare *'four tenths' and 'hundredths'* If teachers are still hesitating writing and comparing decimal numbers, how can they teach learners to correctly read, write and compare decimal numbers in figures and in words?

4.1.3. Doing mathematics operations on decimal numbers

Another key competence targeted in the CBC is having primary school learners able to multiply, add and subtract decimal numbers (REB, 2015a). Fig. 3 describes MSSLs' ability to perform mathematics operations on decimal numbers (on the next page).

(a) How do you write decimals? (b) Four tenths are the same as forty hundredths

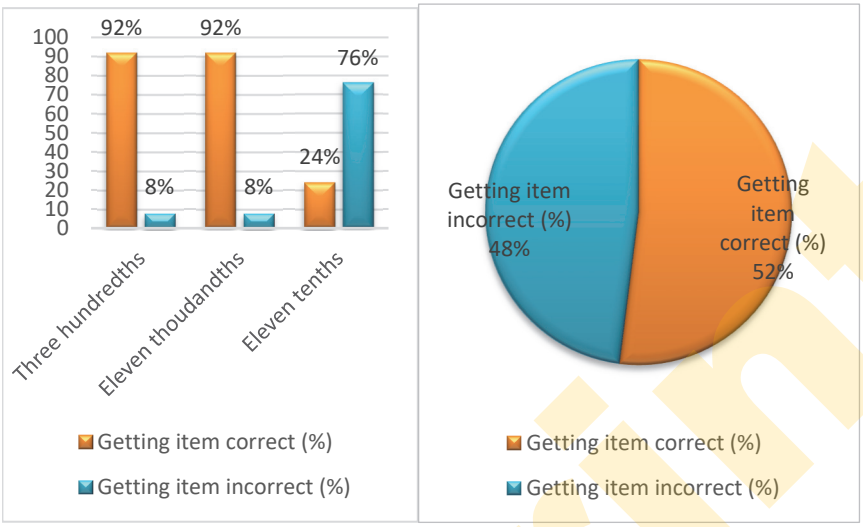

Fig. 2. Write decimal numbers

Though, teachers who plan to teach these operations need to know a lot more than what the CBC foresees for primary school learners (Ball, 1990), Fig. 3 indicates that MSSLs (95%) equally performed in transforming decimal numbers to fractions and in doing addition with decimal numbers. Likewise, MSSLs (86%) did 4 digits subtraction. However, only 43% of MSSLs managed to multiply a decimal number by 100.

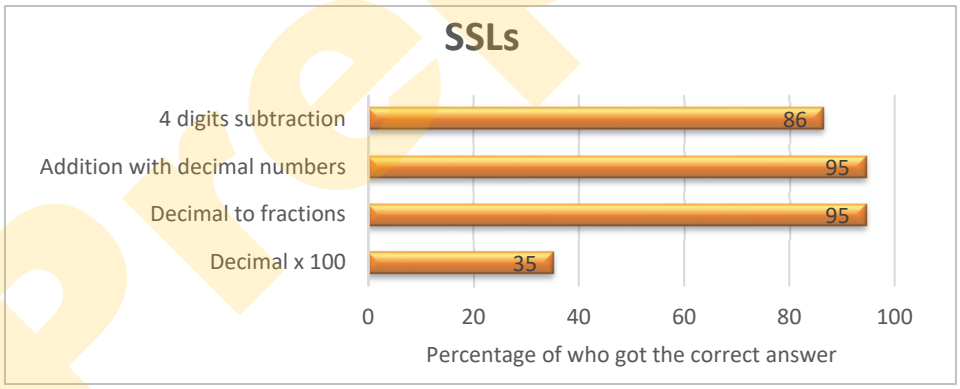

Fig. 3. Mathematics operations on numbers

4.1.4. Rounding of decimal numbers

By the end of primary schools, learners should be able to round off decimals, convert fraction to decimals and *vice versa*. They should be able to solve problems involving rounding and conversion (REB,2015a). MSSLs were asked to identify nearest numbers in size by estimation. Fig. 4 summarizes the percentage of those who correctly answered questions related to rounding of decimal numbers.

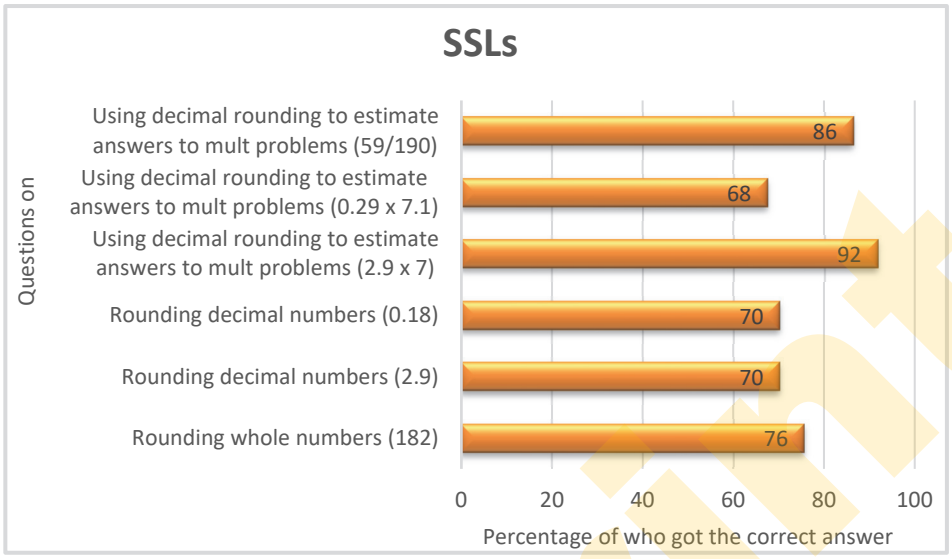

Fig. 4. Using decimal rounding off

Fig. 4 indicates that shows sign of facing difficulties in using decimal rounding to estimate answers to multi-problems, estimating the number to nearest in size to the answer, multiplying with decimal numbers. In addition, only 70% of MSSLs could identify a nearest number in size to 0.18. Again, this shows some weaknesses in manipulating decimal numbers. In general, MSSLs showed weaknesses in rounding off decimal numbers to the nearest tens and hundredths.

4.1.5. Selecting the correct operation for word problems

By the end of primary education, the CBC emphasizes that learners should be able to use numbers and operations to solve real problems (REB, 2015). Fig. 5 describes the

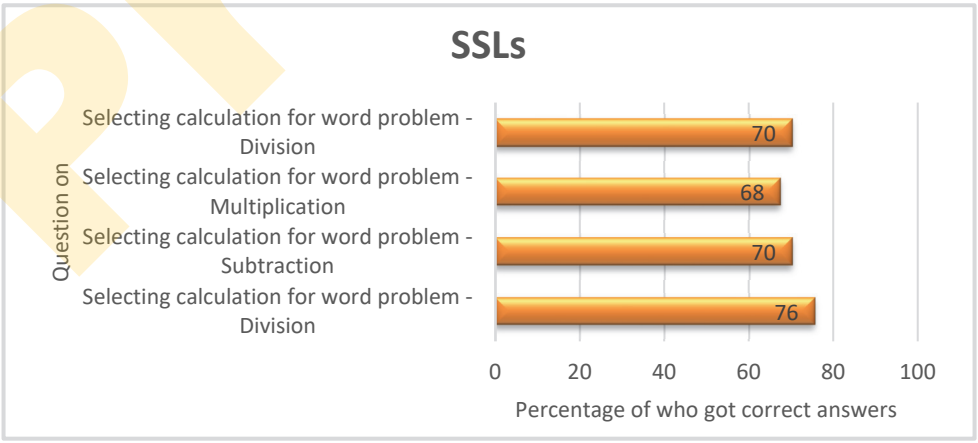

Fig. 5. Selecting calculation for word problem

percent of SSLs who could correctly answer questions that involved addition, multiplication, subtraction and division of whole and decimal numbers.

Fig. 5 indicates that selecting calculation for word problem was not perfect for MSSLs. For a word problem related to addition, they (70%) did it relatively better than they (68%) did for a word problem related to multiplication. It can be inferred that MSSLs need to develop skills related to calculation for word problem which they are teaching.

4.1.6. Division of decimals

As the current curriculum, primary school teachers of mathematics should be able to teach division of numbers. Therefore, they should be conversant with the process of division of decimals up to 3 decimal places. In this view, SSLs were asked a question on dividing two decimal numbers:

12.3 ÷ 0.15 has the same answer as	
A 123 ÷ 0.015	B 123 ÷ 1.5
C 123 ÷ 15	D 123 ÷ 150

Their performance on this question is summarized in the following Fig. 6.

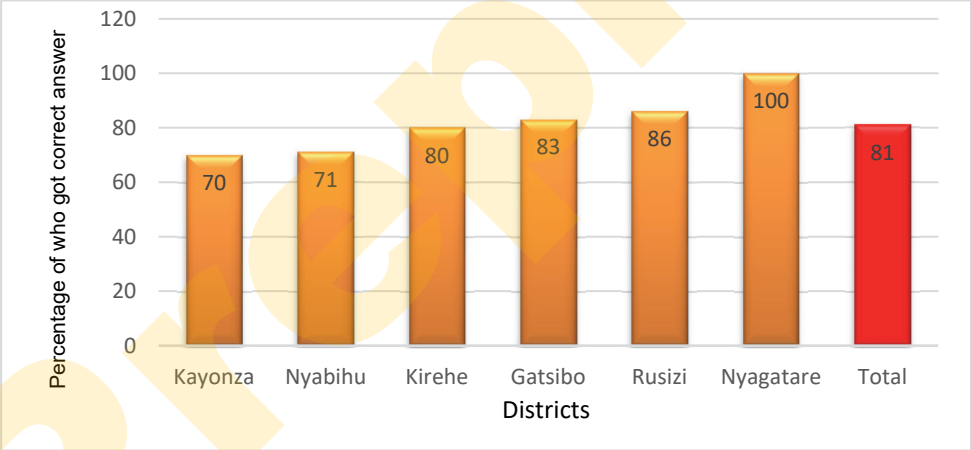

Fig. 6. Performance in division of decimals

Fig. 6 shows that the score is not constant across a given province as Kayonza and Nyagatare located in the same eastern province scored the least and the most respectively.

4.2. Topic area 2: fractions and proportional reasoning

4.2.1. Estimating whole and decimal numbers on a number line

In the CBC framework, teacher of mathematics should be able to involve learners in activities to solve problems related to comparing, ordering, and finding distance between numbers.

From Fig. 7 we can observe that estimating decimal numbers on a number line was more difficult than estimating whole numbers. For examples, only 44% of male SSLs and tutors could estimate a decimal value on an empty interval number line. In addition, only 56% of male SSLs and tutors could estimate decimal value on a grouped interval number line.

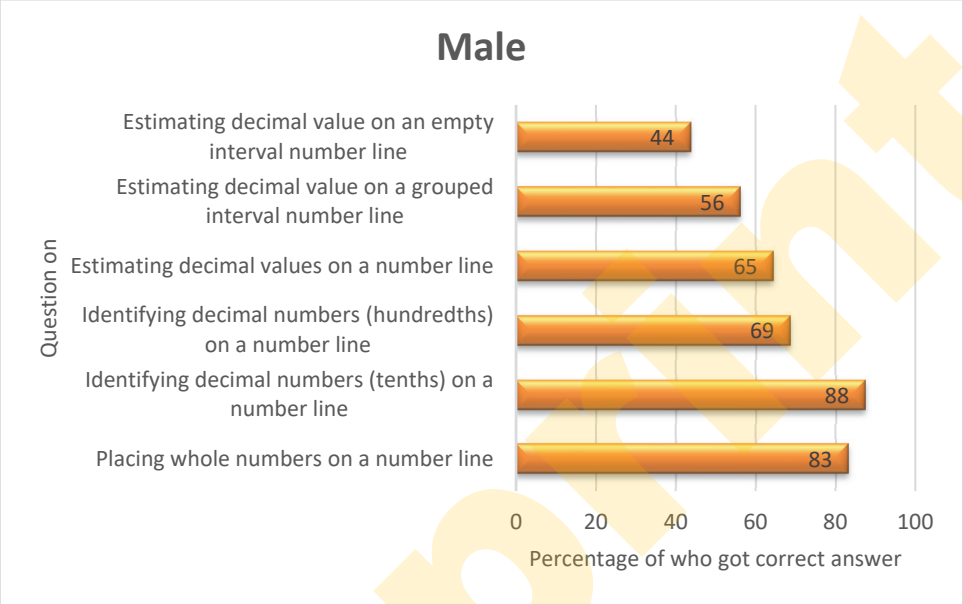

Fig. 7. Estimating whole and decimal numbers on a number line

4.2.2. Proportions and percentage

According to the CBC, teachers should be able to help learners solve simple problems involving proportions, ratios, percentages, mixtures, fractions and decimals (REB, 2015a). As such they should know more on this concept. MSSLs attempted questions on proportions and percentage. Correct answers are summarized in Fig. 8 (on the next page).

Fig. 8 shows that calculating prices after percentage increase and/or reduction was the most challenging activity for teachers and tutors of mathematics in all districts. This was more challenging in Rusizi district where only 50% of teachers and tutors of mathematics could get the right answer. On this activity, all teachers (100%) in Kirehe and Nyagatare did correctly the given question.

4.2.3. Fractions

According to the CBC, teachers and tutors of mathematics in primary schools, should be able to teach learners how to apply fractions in daily life situations and solve related problems (REB, 2015a). As such, they should know more than what learners expect from them. Fig. 9 (on the next page) summarizes correct answers on a question related to manipulating fractions.

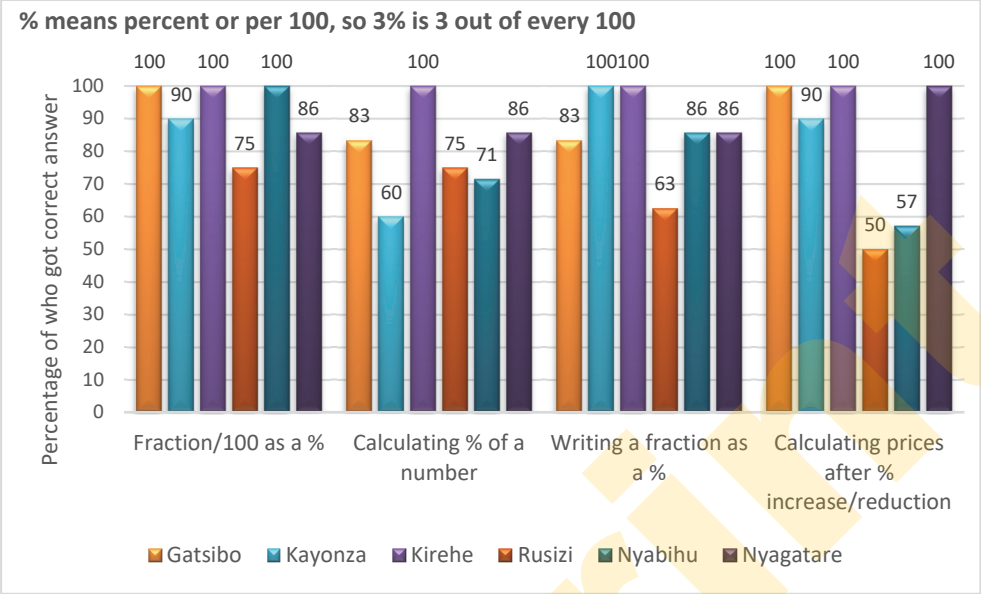

Fig. 8. Word problem on proportions and percentage

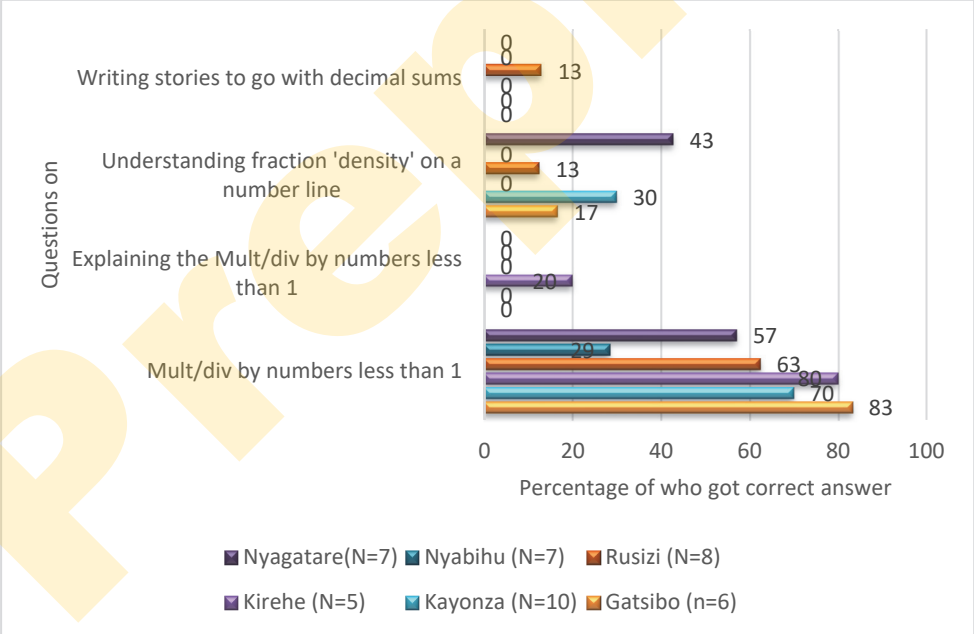

Fig. 9. Working on fractions

Fig. 9 shows that writing stories around decimal sums was challenging for most of teachers. All teachers in Nyagatare, Nyabihu, Kirehe, Kayonza and Gatsibo failed to write an appropriate story around the sum $6.4 + 2.3 = 8.7$.

4.3. Topic area 3: Algebra

4.3.1. Algebraic expressions

Primary school teachers of mathematics should be able to guide students on how to perform algebraic expression (MINEDUC, 1985a). So, teachers should have sufficient knowledge and skills to perform related activities. The following figure summarizes correct answers of teachers and tutors on questions about performing algebraic expressions.

> Question: Chip packets cost R8 each and tins of soup cost R6 each. If c stands for the number of chips packets bought and t stands for the number of tins of soup bought.

From Fig. 10 we can observe that in all districts, performing algebraic expression seemed challenging for teachers of mathematics. The situation showed even worse in Rusizi district where all teachers failed to answer the two questions. Solving word problems that involve simple algebraic equation with two unknown was very challenging for teachers and tutors. Specifically, only 14% and 20% of teachers and tutors in Nyagatare and Kirehe districts respectively could get a write answer to $8c + 6t$.

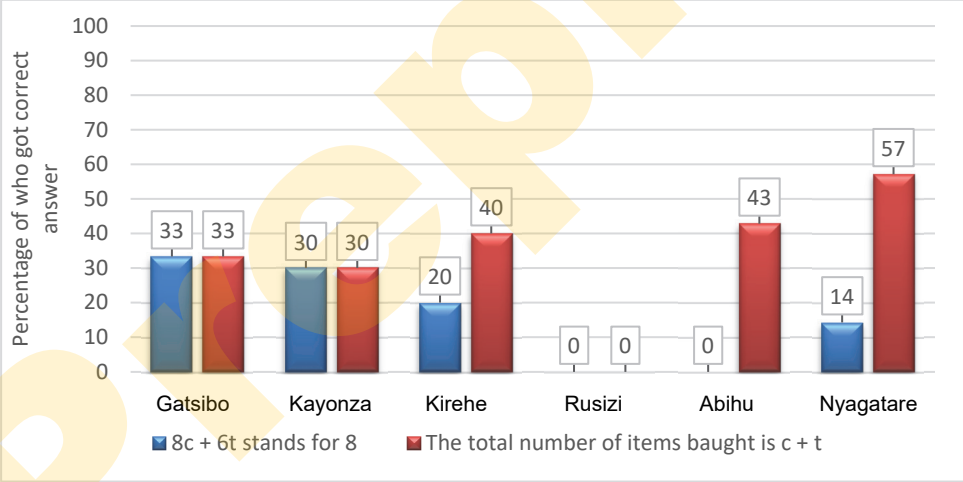

Fig. 10. Performing algebraic expression

4.3.2. Numbers and patterns

According to the CBC, by the end of primary education, learners should be able to describe and generate number patterns following a rule (REB, 2015a). As such, teachers of mathematics in primary schools should know how to determine the pattern for a given expression. They should be able to give learners mathematical word problems to solve by using algebraic methods.

From Fig. 11 we can observe that teachers of mathematics found it difficult to understand repeating pattern structure as well to continue growth patterns from a given examples. Changes are more observed for teachers in Rusizi district than in other districts where only 13% of teachers could correctly answer the question. Furthermore, it was not easy for teachers to extend number patterns to sequences with regularly changing differences. For example, all teachers (100%) in Rusizi district could not express the general term in a growth pattern. In all districts, teachers (100%) failed to provide an equivalent expression when given a machine where to feed numbers to pass out answers.

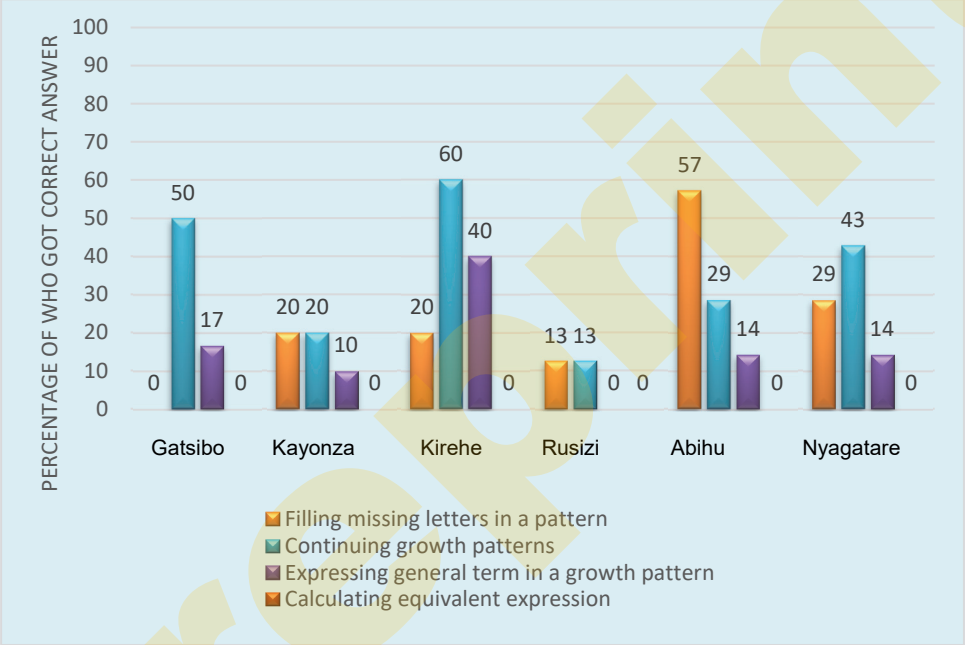

Fig. 11. Determining a pattern for a given expression

4.3.3. Equivalent expressions and number sequences

It is expected that by the end of primary educations, learners should be able to perform operations on algebraic expressions. They should be able to calculate the n^{th} term in a linear expression (REB, 2015a). Therefore, teachers and tutors of mathematics should know more about equivalent expressions and number sequences. In this view, teachers and tutors of mathematics were asked a question to "write down the smallest and the largest number in a number sequence. Correct answers are summarized in Fig. 12 below. Participants were asked to write down the smallest and the largest of these numbers: $n+1,\ n+4, n-3, n, n-7.$

Fig. 12 indicates that all teachers of mathematics in Gatsibo district (100%) could correctly compare relative size of different linear expressions. In other districts some teachers failed to compare the relative size of different linear expressions. For example,

in Kayonza, only 73% of teachers could identify down the smallest and the largest number in sequence.

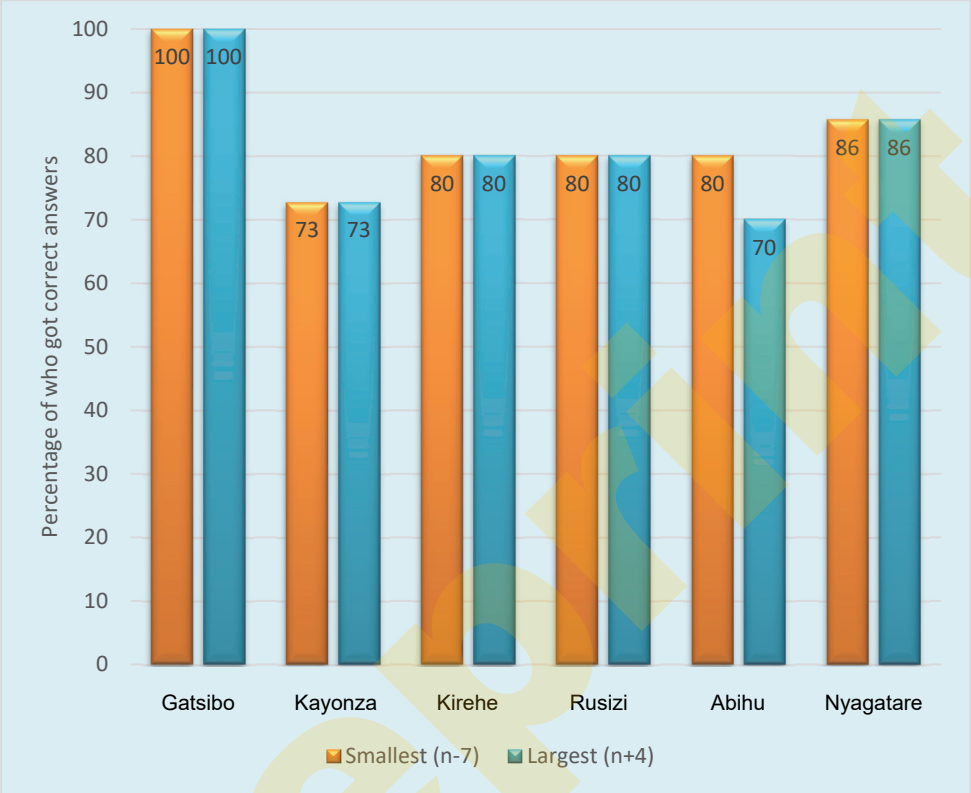

Fig. 12. Comparing the relative size of different linear expressions

4.4. Topic area 4: Geometry

Teachers of mathematics have to teach learners how to find different dimensions of geometrical shapes and to solve mathematical problems related to geometrical figures as well as recognizing special quadrilaterals.

4.4.1. Calculating rectangle area and perimeter

According to the CBC teachers of mathematics in primary school should be able to show the origin of formulae and how to use them to calculate area and perimeter of a regular polygon (REB, 2015a). In this view MSSLs were asked to calculate the area and perimeter of a rectangle. Their correct answers are summarized in Figure 13 below. Participants were given different shapes an asked to estimate areas and perimeters.

From Fig. 13 one can observe that rectangle perimeter and area as an algebraic expression was the most challenging activities for teachers. For example, only 14% of teachers and tutors of mathematics could calculate the rectangle perimeter as an

algebraic expression of one variable. On the other hand, only 28% of them were able to calculate the rectangle area as an algebraic expression of one variable. Calculating rectangle area given measurements seemed easier than calculating its perimeter. For example, about 96% of teachers and tutors of mathematics could calculate the rectangle area given measurement, whereas only 68% of them were able to calculate the rectangle perimeter given measurement.

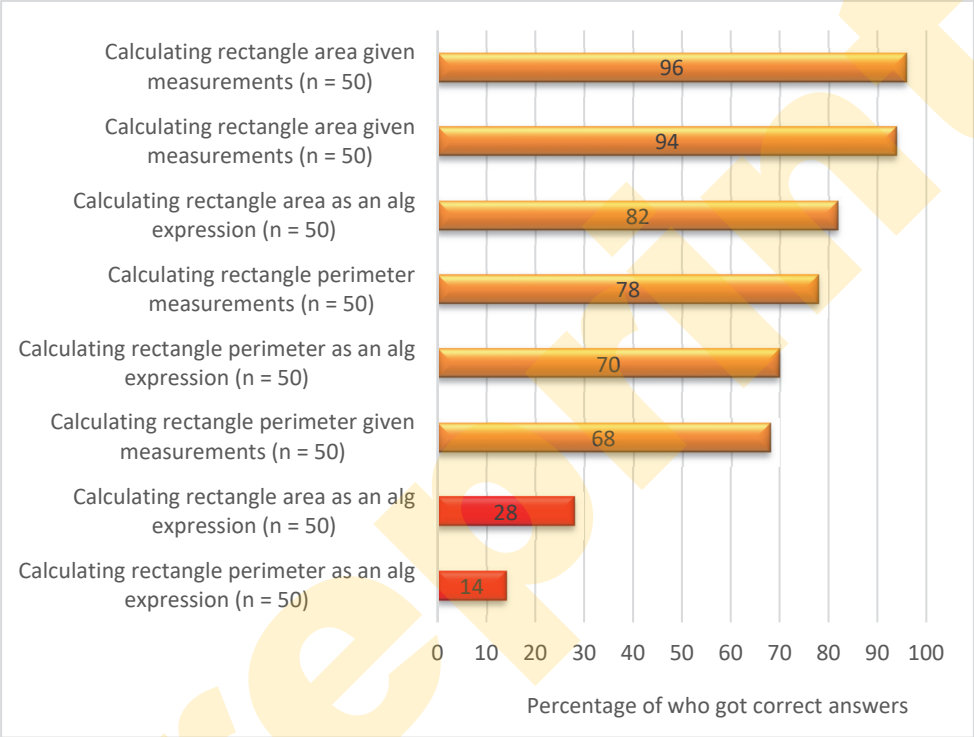

Fig. 13. Calculating rectangle area given measurements

In general, teachers of mathematics lacked basic knowledge on calculating rectangle areas and perimeter. This was more difficulty for calculations that involved. algebraic calculation. Therefore, they need more activity to improve on this knowledge so that they can effectively teach these concepts in primary schools.

4.4.2. Operations on geometry

Whereas teachers of mathematics should teach learners how to solve mathematical problems related to the finding the volume of cuboid and cubes, only 40% of teachers were able to solve a word problem that involved the relationship between cubic cm and liter. In addition, only 66% of mathematics teachers and tutors could be able to identify a trapezium from other shapes such as parallelogram, quadrilateral, rhombs and square.

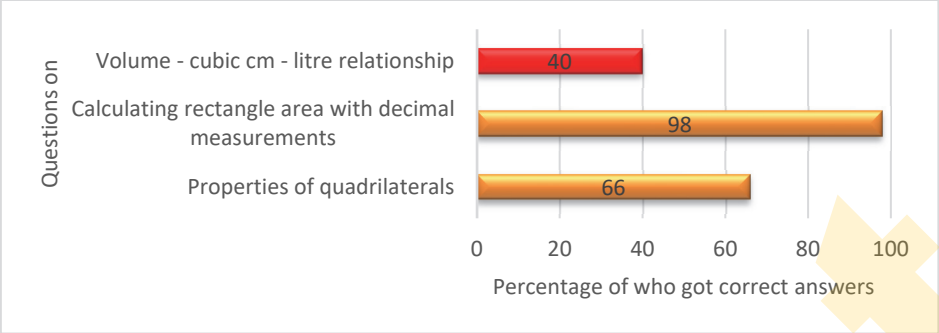

Fig. 14. Working out operations on geometry

4.5. Topic area 5: Statistics and elementary probability

By the end of primary school learners should be able to collect, represent and interpret data (REB, 2015). As such teachers of mathematics should know more on how to collect, represent and interpret data. They were asked the following question:

> in a survey, 40 parents were asked how many children they have. Provided number to present and interpret data.

Percentages of teachers who got correct answers are presented in Fig. 15.

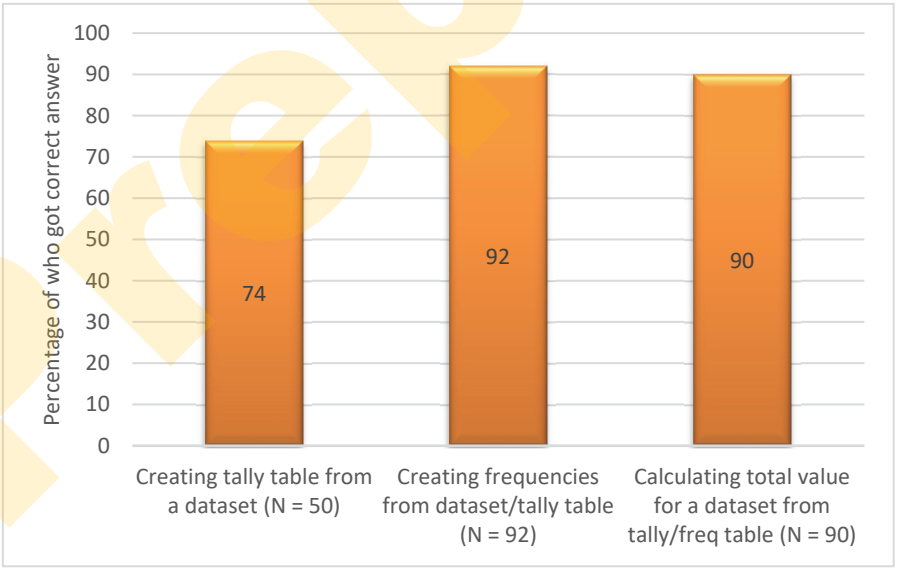

Fig. 15. Data collection, presentation and interpretation

From Fig. 15 we can observe that 74% of teachers of mathematics could create correctly tally table from a data base. At the same time 92% of teachers could create

frequencies from dataset/tally; and 90% of them could calculate correctly the total value for a dataset from tally.

4.6. Topic Area 6: Measurement

4.6.1. Measuring time

By the end of primary education, learners should be able to solve problem involving time interval (REB,2015a). As such their teachers should know and be able to teach how to solve real life problems that involve finding time intervals and conversion. Teachers were asked to solve the problem: *A tray of meringues is placed in the oven at 7:40. The meringues need to bake at a low temperature for 2.5 hours. At what time must they be taken out of the oven?* Figure 16 below represents the percentage of those who got correct answer per district.

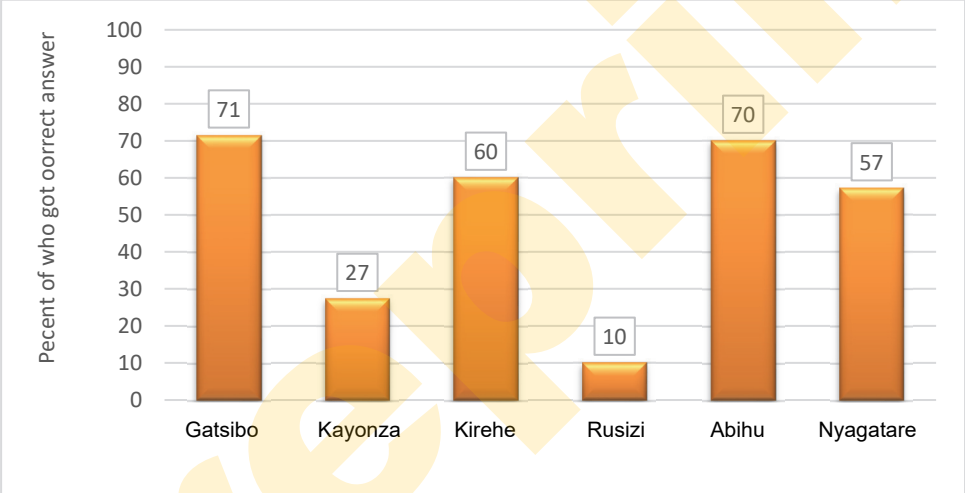

Fig. 16. Calculating time duration

Some teachers failed to calculate time duration with time given in decimal format. In Kayonza and Rusizi district, only 27% and 10% of mathematics teachers could provide correct answers.

4.6.2. Measuring capacity

According to the CBC, by the end of primary education, learners should be able solve mathematical problems involving capacity measurement (REB, 2015). As such teachers of mathematics should know more on how to explain the conversion of units and how to do multiplicative scale up and down.

> One litre of petrol costs R10.75: Provide an answer with a method AND an explanation for working out the costs of (i) 0.53 litre (ii) 3 litres.

Fig. 17 shows that mathematics teachers face difficulties to explain the multiplicative scale up and down operations involving units of capacity: only 20% of teachers could attach a correct explanation to the multiplication they are able to do; only 60% and 70% of teachers could do the multiplicative scale up down and up respectively.

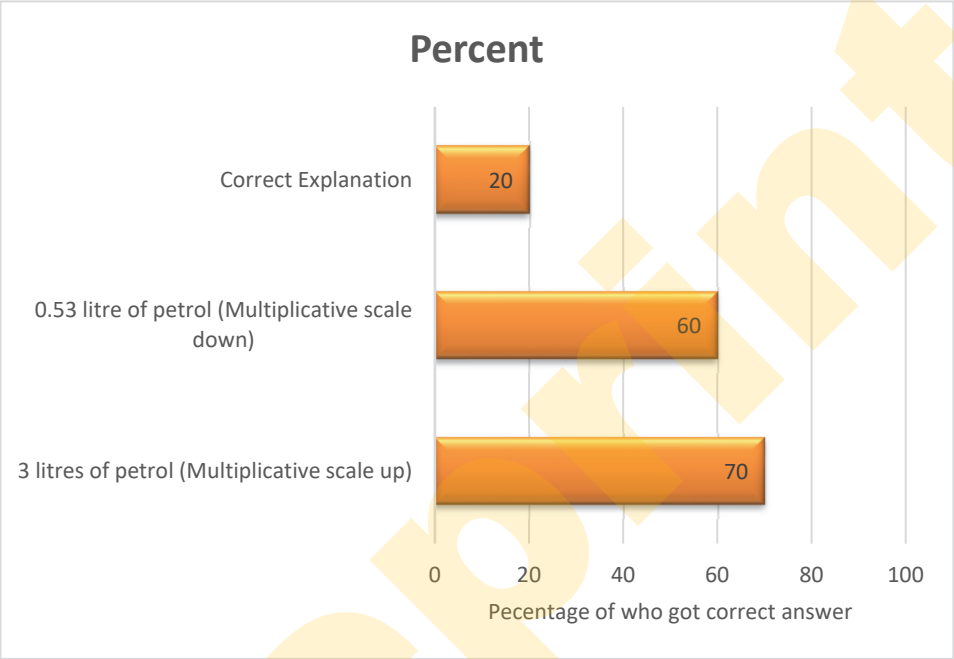

Fig. 17. Provide an answer with a method

4.6.3. Measuring the lengths

Measuring lengths is an important competence targeted for learners by the end of their primary educations. Specifically, as per CBC, learners should be able to convert between units of lengths and apply them in solving mathematical problems related to daily life situations (REB, 2015a).

> When we meaure ther heights with matchsticks, Mr Short's height is four matchsticks. Mr Tall's height is six matchsticks. How many paper clips do we need for Mr Tall's height.

While teachers of mathematics should teach about proportional reasoning, most of them do not have knowledge to manipulate ratios as shown by Fig. 18. In particular, all teachers in Rusizi, could not provide correct answer on determining the ratio of one unit of measurement in terms of another.

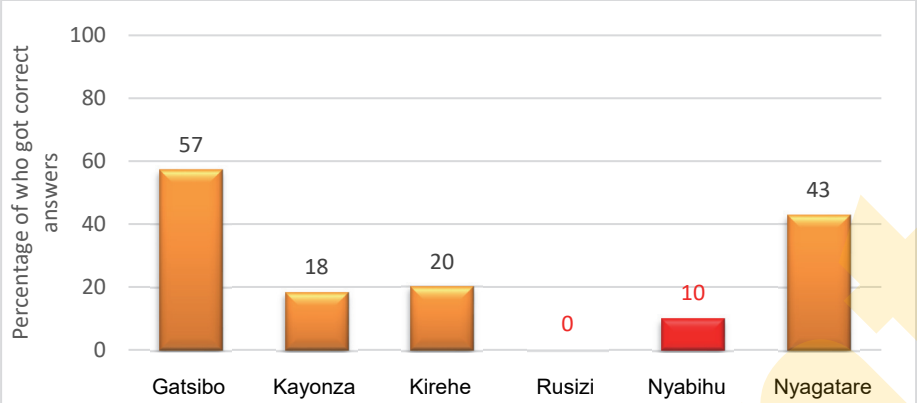

Fig. 18. Ratio

4.7. General performance per district

Fig. 19 below presents general performance by district in terms of percentage of teachers who got correct answers to the 20 questions.

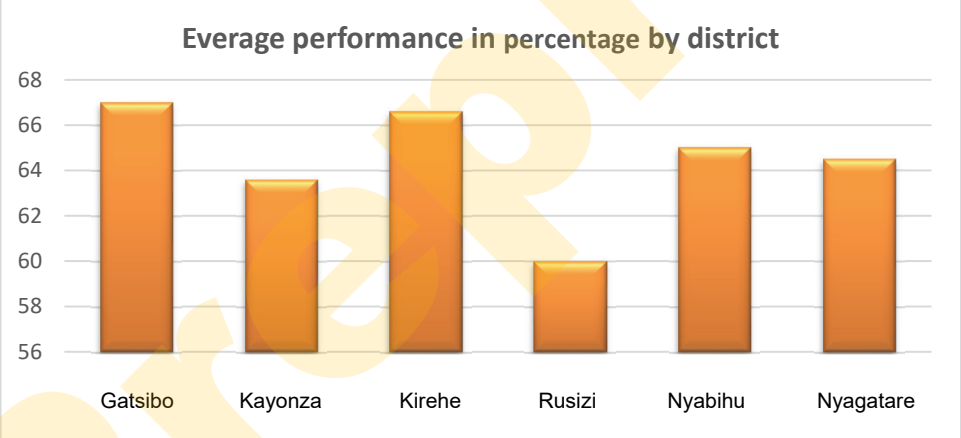

Fig. 19. Average performance per district

Fig. 19 shows that the general performance of mathematics teachers in the test-ranged from 60 to 67 percent; the highest average performance is observed in Gatsibo district where about 67% of teachers could give correct answers to the pre-test questions and the lowest average performance is observed in Rusizi district with 60%.

4.8. General performance per topic area

Across grades of primary education, learner learn the content divided into different topic areas such as (1) numbers and operations, (2) fractions, decimals and proportional reasoning, (3) Algebra, (4) Geometry, (5) statistics and elementary probability and (6) measurement (MINEDUC, 2015).

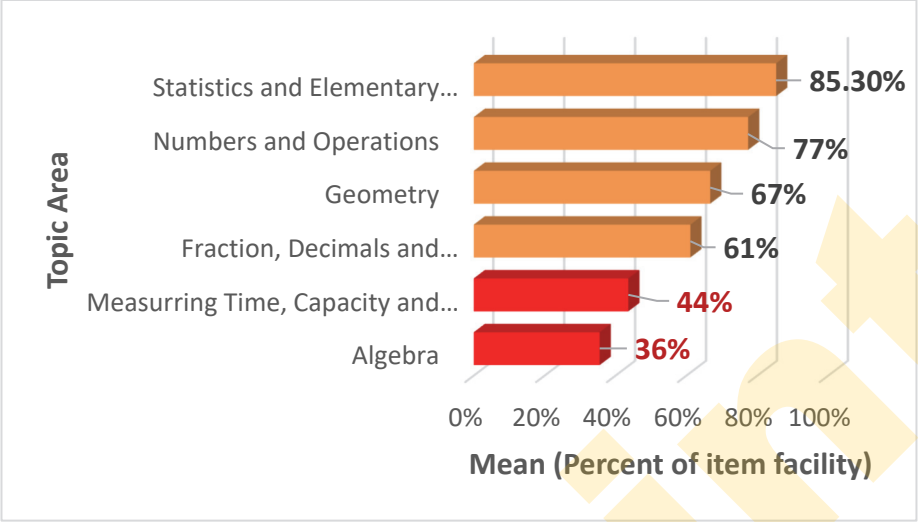

Fig. 20. Average Performance per topic area

The performance in terms of item facility varied across topic areas. This implies that teachers and tutors of mathematics did not have same comprehensive knowledge in all six topic areas indicated in the CBS's syllabus. Mathematics teachers had difficulties in performing on questions related to Algebra and Measurement. On average, only 36% of respondents could perform correctly item related to algebra whereas only 44% could get correct answers for items related to measurement. Statistics and probability seemed the easiest topic area to be performed by teachers and tutors of mathematics since 85% could perform correctly the question related to statistics.

5. Discussions and Conclusions

The Rwanda competence-based curriculum framework (REB, 2015) places numeracy as one of the seven basic competences that are required for all children from preprimary education. Furthermore, literacy and numeracy are considered as basic to accessing learning in other subjects. With regard to numeracy, it is expected that by the end of primary education, all children must be equipped with skills in computing accurately using the four mathematical operations, manipulating numbers, mathematical symbols, quantities, shapes and figures to accomplish a task involving calculations, measurements and estimations. In addition, they should be able to use numerical patterns and relations to solve problems related to everyday activities like commercial context and financial management as well interpreting basic statistical data using tables, diagrams, charts and graphs (REB,2015). However, despite this emphasis, observations on the ground and national examination tests reveal that children are not yet facilitated to achieve these objectives. Part of the present paper was to explore to what extent teachers are themselves conversant with the content. Though the study

used a small number of participants, results can help to rethink the focus of CPD activities and start to think about the link of learners' failure with teachers' content knowledge.

The general consideration shows that MSSLs do face challenges in all topics of the mathematics syllabus while those from Rusizi seem to be the most challenged. For example, understanding fraction 'density' on a number line was challenging for most of teachers and tutors of mathematics. Also, all teachers in Nyabihu, and Kirehe failed to tell that between ¼ and ½ there are infinite number (many) of fractions. In all district, MSSLs could work out the multiplication with a number less than 1, most of them could not explain why dividing 72.4 by 8/7 you get a bigger answer than multiplying the same number by 7/8. However, teachers in Nyabihu district were less competent to perform this activity; all teachers in Nyagatare, Kayonza, Rusizi, Nyabihu and Gatsibo could not give correct explanations.

If the mathematics content knowledge is at lower level, it is hypothetically expected that teachers will not be confident in the teaching, thus hindering children' s learning. It therefore deductible that low level of performance from pupils in the sampled region might have partial explanations in low teacher' s mastery of the subject knowledge.

This shows that their knowledge on algebraic expression needs to be sharpened so that they can teach effectively algebra in primary school. Therefore, teachers of mathematics in primary schools needs a training on the knowledge related to equivalent expressions and number sequence. This implies that they could correctly teach the concept of equivalent expressions and number sequences.

Though MSSLs performed much better in the topic area of statistics and probability, this can not lead us to confirm that the topic is well taught or understood since there was only one question of statistics without any question related to probability. A parallel study (Dushimana and Uworwabayeho, 2021) that is analyzing preservice primary school teachers' performance in national mathematics exams for the period of 2014-2016 shows that the questions related to this topic are the most failed in addition to register general failure in mathematics.

Therefore, we can deduct a lack of teachers' preparedness to adopt the new curriculum teaching approaches, there is also lack of appropriate physical facilities in schools to accommodate every leaner's individual needs among other hindering factors. Strengthening PCK is a key instrument to improve the quality of teaching and learning. PCK develops with teaching experience. However, it doesn't come automatically, but requires continuous professional development and reflection. Through observations, constructions, hands-on manipulations, generalisations, and presentations of information during the learning process, the learner will not only develop deductive and inductive skills but also acquire co-operation, communication, critical thinking and problem-solving skills. This will be realized when learners make presentations leading to inferences and conclusions at the end of the learning unit. This will be achieved if teachers are capable to embed different teaching strategies ranging from simple

operations to problem solving through converting fractions into decimals and vice versa, writing numbers in figures and reciprocally.

Recommendations include systematic CPD programmes for in service teachers to complement preservice training so that they can adapt various reforms and in-service teachers to establish their individual professional development plans. As for further area for exploration, we would suggest to extend this research by assessing the impact of the on learners achieving learning outcomes and further issues such as reducing drop outs and improving girls' performance in mathematics.

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